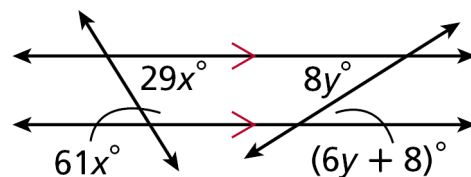


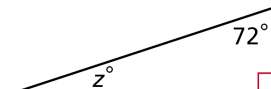
**Attendance Problems.** Find the value of each variable.

1.  $x$

2.  $y$



3.  $z$



- I can prove and apply properties of parallelograms.
- I can use properties of parallelograms to solve problems.

**Common Core CC.9-12.G.CO.11** Prove theorems about parallelograms.

Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names.

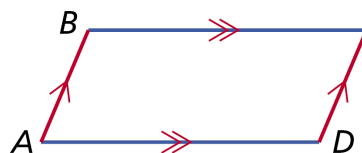
## Helpful Hint

Opposite sides of a quadrilateral do not share a vertex. Opposite angles do not share a side.

A quadrilateral with two pairs of parallel sides is a **parallelogram**. To write the name of a parallelogram, you use the symbol  $\square$ .

Refer to  
page 404.

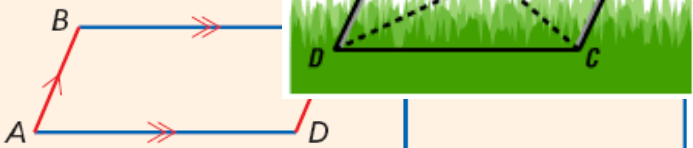
Parallelogram  $ABCD$   
 $\square ABCD$



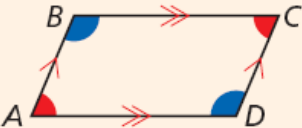
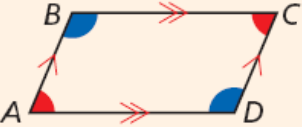
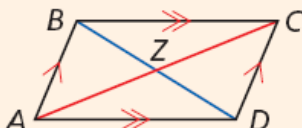
Example 1 on

$$\overline{AB} \parallel \overline{CD}, \overline{BC} \parallel \overline{DA}$$

**Theorem 6-2-1** Properties of Parallelograms

THEOREM	HYPOTHESIS
<p>If a quadrilateral is a parallelogram, then its opposite sides are congruent.</p> <p>(<math>\square \rightarrow \text{opp. sides} \cong</math>)</p>	

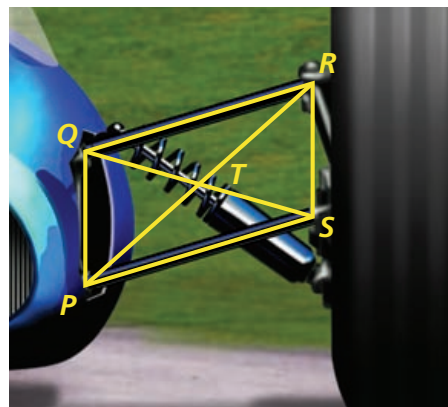
**Theorems** Properties of Parallelograms

THEOREM	HYPOTHESIS	CONCLUSION
<p><b>6-2-2</b> If a quadrilateral is a parallelogram, then its opposite angles are congruent.</p> <p>(<math>\square \rightarrow \text{opp. } \angle \cong</math>)</p>		$\angle A \cong \angle C$ $\angle B \cong \angle D$
<p><b>6-2-3</b> If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.</p> <p>(<math>\square \rightarrow \text{cons. } \angle \text{ supp.}</math>)</p>		$m\angle A + m\angle B = 180^\circ$ $m\angle B + m\angle C = 180^\circ$ $m\angle C + m\angle D = 180^\circ$ $m\angle D + m\angle A = 180^\circ$
<p><b>6-2-4</b> If a quadrilateral is a parallelogram, then its diagonals bisect each other.</p> <p>(<math>\square \rightarrow \text{diags. bisect each other}</math>)</p>		$\overline{AZ} \cong \overline{CZ}$ $\overline{BZ} \cong \overline{DZ}$

**Video Example 1.** An inexpensive picnic table begins to lean over and forms a parallelogram. The diagram shows the picnic table. In  $\square ABCD$ ,  $AD = 60$  inches,  $AE = 35$  inches, and  $m\angle CDA = 81^\circ$ . Find each measure.

**1 Racing Application**

The diagram shows the parallelogram-shaped linkage that joins the frame of a race car to one wheel of the car. In  $\square PQRS$ ,  $QR = 48$  cm,  $RT = 30$  cm, and  $m\angle QPS = 73^\circ$ . Find each measure.



**A**  $PS$   
 $\overline{PS} \cong \overline{QR}$   $\square \rightarrow \text{opp. sides} \cong$   
 $PS = QR$   $\text{Def. of } \cong \text{ segs.}$   
 $PS = 48$  cm  $\text{Substitute 48 for QR.}$

**B**  $m\angle PQR$   
 $m\angle PQR + m\angle QPS = 180^\circ$   $\square \rightarrow \text{cons. } \angle \text{ supp.}$   
 $m\angle PQR + 73 = 180$   $\text{Substitute 73 for } m\angle QPS.$   
 $m\angle PQR = 107^\circ$   $\text{Subtract 73 from both sides.}$

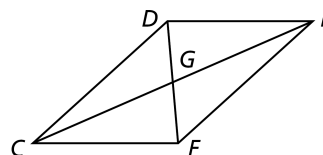
**C**  $PT$   
 $\overline{PT} \cong \overline{RT}$   $\square \rightarrow \text{diags. bisect each other}$   
 $PT = RT$   $\text{Def. of } \cong \text{ segs.}$   
 $PT = 30$  cm  $\text{Substitute 30 for RT.}$

**Example 1.** In  $\square CDEF$ ,  $DE = 74$  mm,  $DG = 31$  mm, and  $m\angle FCD = 42^\circ$ . Find each measure.

A. CF

B.  $m\angle EFC$ 

C. DF

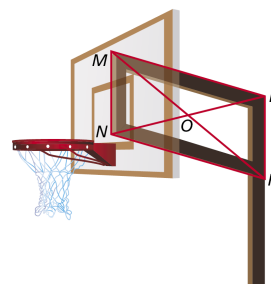


**Guided Practice.** In  $\square KLMN$ ,  $LM = 28$  in.,  $LN = 26$  in., and  $m\angle LKN = 74^\circ$ . Find each measure.

4. KN

5.  $m\angle NML$ 

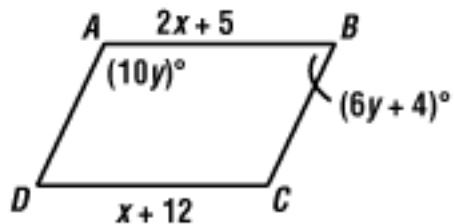
6. LO



**Video Example 2.** Figure ABCD is a parallelogram. Find each measure.

A. AB

B.  $m\angle B$



**2**

## Using Properties of Parallelograms to Find Measures

ABCD is a parallelogram. Find each measure.

**A**  $\overline{AD}$

$$\overline{AD} \cong \overline{BC}$$

$\square \rightarrow \text{opp. sides} \cong$

$$AD = BC$$

Def. of  $\cong$  segs.

$$7x = 5x + 19$$

Substitute the given values.

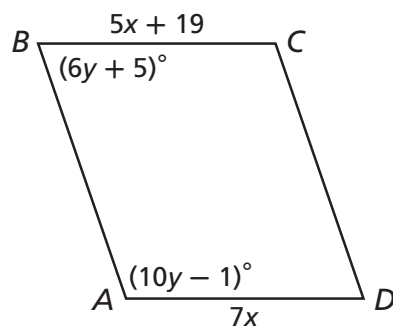
$$2x = 19$$

Subtract  $5x$  from both sides.

$$x = 9.5$$

Divide both sides by 2.

$$AD = 7x = 7(9.5) = 66.5$$



**B**  $m\angle B$

$$m\angle A + m\angle B = 180^\circ$$

$\square \rightarrow \text{cons. } \angle \text{ supp.}$

$$(10y - 1) + (6y + 5) = 180$$

Substitute the given values.

$$16y + 4 = 180$$

Combine like terms.

$$16y = 176$$

Subtract 4 from both sides.

$$y = 11$$

Divide both sides by 16.

$$m\angle B = (6y + 5)^\circ = [6(11) + 5]^\circ = 71^\circ$$

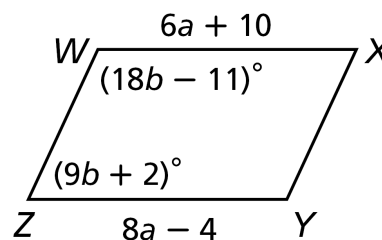
**Question:** What do you call an urgent message sent across a parallel network?

**Answer:** A parallelogram.

**Example 2.**  $WXYZ$  is a parallelogram.

A.  $YZ$

B.  $m\angle Z$

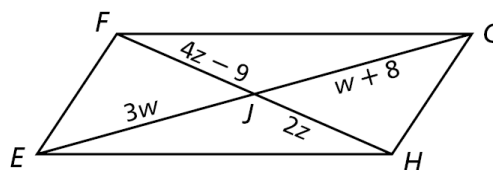


**Guided Practice.**  $EFGH$  is a parallelogram.

Find each measure.

7.  $JG$

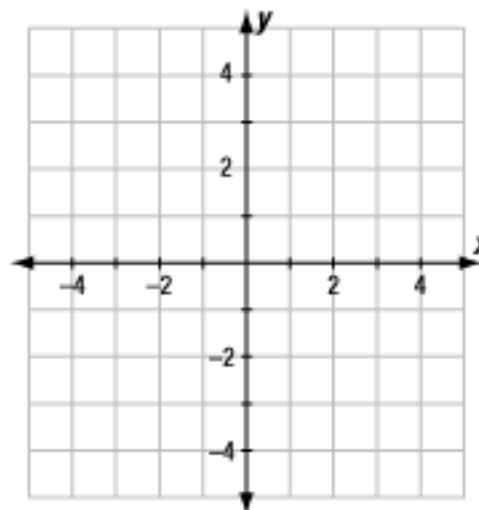
8.  $FH$



## Remember!

When you are drawing a figure in the coordinate plane, the name  $ABCD$  gives the order of the vertices.

**Video Example 3.** Three vertices of a parallelogram are  $A(-3, 3)$ ,  $B(-4, 0)$ , and  $C(2, -4)$ . Find the coordinates of vertex  $D$ .



**3****Parallelograms in the Coordinate Plane**

Three vertices of  $\square ABCD$  are  $A(1, -2)$ ,  $B(-2, 3)$ , and  $D(5, -1)$ . Find the coordinates of vertex  $C$ .

Since  $ABCD$  is a parallelogram, both pairs of opposite sides must be parallel.

**Step 1** Graph the given points.

**Step 2** Find the slope of  $\overline{AB}$  by counting the units from  $A$  to  $B$ .

The rise from  $-2$  to  $3$  is  $5$ .

The run from  $1$  to  $-2$  is  $-3$ .

**Step 3** Start at  $D$  and count the same number of units.

A rise of  $5$  from  $-1$  is  $4$ .

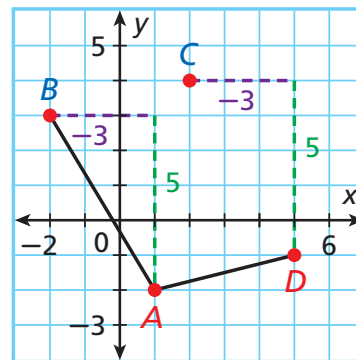
A run of  $-3$  from  $5$  is  $2$ . Label  $(2, 4)$  as vertex  $C$ .

**Step 4** Use the slope formula to verify that  $\overline{BC} \parallel \overline{AD}$ .

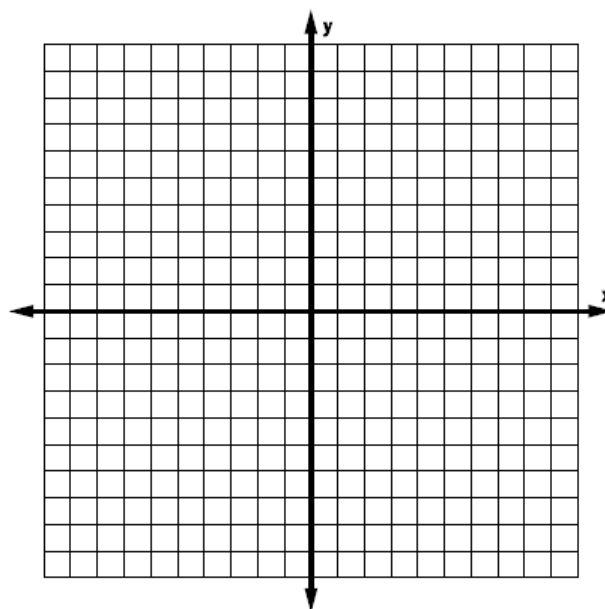
$$\text{slope of } \overline{BC} = \frac{4 - 3}{2 - (-2)} = \frac{1}{4}$$

$$\text{slope of } \overline{AD} = \frac{-1 - (-2)}{5 - 1} = \frac{1}{4}$$

The coordinates of vertex  $C$  are  $(2, 4)$ .

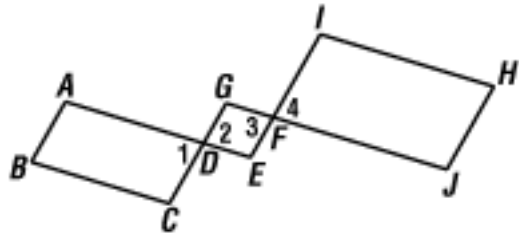


**9. Guided Practice.** Three vertices of  $\square PQRS$  are  $P(-3, -2)$ ,  $Q(-1, 4)$ , and  $S(5, 0)$ . Find the coordinates of vertex  $R$ .



**Video Example 4.**

**Given:** ABCD, DEFG, & FIHJ are parallelograms. A, D, and E are collinear. C, D, and G are collinear. J, F, and G are collinear and E, F, and I are collinear.



**6-2 Properties of Parallelograms** (*p 407*) 17-25 odd, 28, 42, 46-50.